2015年度　数値計算(西谷) 解答例

1 fsolve

```
> restart;
  func:=x->-4*exp(-x)+2*exp(-2*x);
```

$$func := x \rightarrow -4\,e^{-x} + 2\,e^{-2x} \qquad (2.1)$$

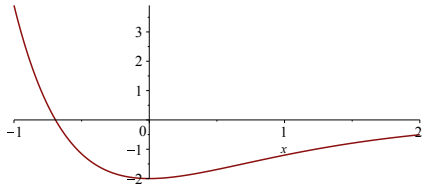
```
> solve(func(x),x);
```

$$-\ln(2) \qquad (2.2)$$

```
> x0:=evalf(-log(2));
```

$$x0 := -0.6931471806 \qquad (2.3)$$

```
> plot(func(x),x=-1..2);
```




```
> x1:=-1.0: x2:=0.0: res1:=[];
>   f1:=func(x1): f2:=func(x2):
>   for i from 1 to 10 do
>     x:=(x1+x2)/2;
>     f:=func(x);
>     if f*f1>=0.0 then
>       x1:=x; f1:=f;
>     else
>       x2:=x; f2:=f;
>     end if;
>     printf("%20.15f, %20.15f\n",x,f);
  res1:=[op(res1),[i,abs(x-x0)]];
>   end do:
```

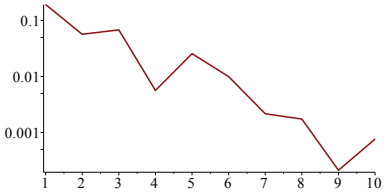
$$res1 := [\,]$$

```
  -0.500000000000000,   -1.158321428000000
  -0.750000000000000,    0.495378072000000
  -0.625000000000000,   -0.492297914000000
  -0.687500000000000,   -0.044796434000000
  -0.718750000000000,    0.212847420000000
  -0.703125000000000,    0.081026558000000
  -0.695312500000000,    0.017378914000000
  -0.691406250000000,   -0.013891124000000
  -0.693359375000000,    0.001698096000000
  -0.692382812500000,   -0.006107938000000
```

```
> res1;
```

```
  with(plots):
  logplot(res1);
```

[[1, 0.1931471806], [2, 0.0568528194], [3, 0.0681471806], [4, 0.0056471806], [5, 0.0256028194], [6, 0.0099778194], [7, 0.0021653194], [8, 0.0017409306], [9, 0.0002121944], [10, 0.0007643681]]



2 round error

```
> restart;
  den:=23.173-23.094;
  num:=0.81321;
  num/den;
```

$$den := 0.079$$
$$num := 0.81321$$
$$10.29379747 \qquad (3.1)$$

```
> Digits:=5;
  den:=23.173-23.094;
  num:=0.81321;
  num/den;
```

$$Digits := 5$$
$$den := 0.079$$
$$num := 0.81321$$
$$10.294 \qquad (3.2)$$

```
> Digits:=4;
  den:=23.173-23.094;
  num:=0.81321;
  num/den;
```

$$Digits := 4$$
$$den := 0.08$$
$$num := 0.81321$$
$$10.16 \qquad (3.3)$$

```
> Digits:=3;
  den:=23.173-23.094;
  num:=0.81321;
  num/den;
```

$$Digits := 3$$
$$den := 0.1$$
$$num := 0.81321$$
$$8.13 \quad (3.4)$$

```
> Digits:=2;
  den:=23.173-23.094;
  num:=0.81321;
  num/den;
```

$$Digits := 2$$
$$den := 0.$$
$$num := 0.81321$$
$$Float(\infty) \quad (3.5)$$

3 Newton's interpolation

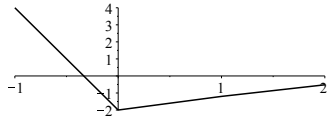
```
> restart;
  func:=x->-4*exp(-x)+2*exp(-2*x);
  X:=[-1,0,1,2];Y:=[];
  for i from 1 to 4 do
  Y:=[op(Y),evalf(func(X[i]),2)];
  end;
```

$$func := x \rightarrow -4\,e^{-x} + 2\,e^{-2x}$$
$$X := [-1, 0, 1, 2]$$
$$Y := [\,]$$
$$Y := [4.]$$
$$Y := [4., -2.]$$
$$Y := [4., -2., -1.2]$$
$$Y := [4., -2., -1.2, -0.52] \quad (4.1)$$

```
> with(LinearAlgebra):
  list1:=[X,Y];
```

$$list1 := [[-1, 0, 1, 2], [4., -2., -1.2, -0.52]] \quad (4.2)$$

```
> with(plots):
  l1p:=listplot(Transpose(Matrix(list1))):
  display(l1p);
```




```
> F2:=y0+(x-x0)*f1_01+(x-x0)*(x-x1)*f2_012;
  F3:=y0+(x-x0)*f1_01+(x-x0)*(x-x1)*f2_012+(x-x0)*(x-x1)*(x-x2)*
  f3_0123;
```

$$F2 := y0 + (x - x0)\,f1\_01 + (x - x0)\,(x - x1)\,f2\_012$$
$$F3 := y0 + (x - x0)\,f1\_01 + (x - x0)\,(x - x1)\,f2\_012 + (x - x0)\,(x - x1)\,(x - x2)\,f3\_0123 \quad (4.3)$$

```
> f1_01:=(y1-y0)/(x1-x0);
  f1_12:=(y2-y1)/(x2-x1);
  f1_23:=(y3-y2)/(x3-x2);
```

$$f1\_01 := \frac{y1 - y0}{x1 - x0}$$
$$f1\_12 := \frac{y2 - y1}{x2 - x1}$$
$$f1\_23 := \frac{y3 - y2}{x3 - x2} \quad (4.4)$$

```
> f2_012:=(f1_12-f1_01)/(x2-x0);
  f2_123:=(f1_23-f1_12)/(x3-x1);
```

$$f2\_012 := \frac{\frac{y2 - y1}{x2 - x1} - \frac{y1 - y0}{x1 - x0}}{x2 - x0}$$
$$f2\_123 := \frac{\frac{y3 - y2}{x3 - x2} - \frac{y2 - y1}{x2 - x1}}{x3 - x1} \quad (4.5)$$

```
> f3_0123:=(f2_123-f2_012)/(x3-x0);
```

$$f3\_0123 := \frac{\frac{\frac{y3 - y2}{x3 - x2} - \frac{y2 - y1}{x2 - x1}}{x3 - x1} - \frac{\frac{y2 - y1}{x2 - x1} - \frac{y1 - y0}{x1 - x0}}{x2 - x0}}{x3 - x0} \quad (4.6)$$

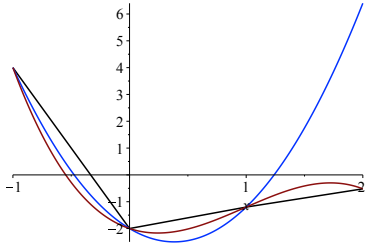
```
> F2;
```

$$y0 + \frac{(x - x0)\,(y1 - y0)}{x1 - x0} + \frac{(x - x0)\,(x - x1)\left(\frac{y2 - y1}{x2 - x1} - \frac{y1 - y0}{x1 - x0}\right)}{x2 - x0} \quad (4.7)$$

```
> for i from 1 to 4 do
    x||(i-1):=X[i];
    y||(i-1):=Y[i];
  end:
> eq2:=F2;
  eq3:=F3;
```

$$eq2 := -2. - 6.\,x + 3.400000000\,(x + 1)\,x$$
$$eq3 := -2. - 6.\,x + 3.400000000\,(x + 1)\,x - 1.153333333\,(x + 1)\,x\,(x - 1) \quad (4.8)$$

```
> with(plots):
  l1p:=listplot(Transpose(Matrix(list1))):
  pf2:=plot(eq2,x=-1..2,color=blue):
  pf3:=plot(eq3,x=-1..2):
  display(l1p,pf2,pf3);
```



4 PageRank

```
> restart;
  AA:=Matrix([[0,1,1,1,0],[1,0,1,0,0],
  [0,0,0,1,0],[0,0,1,0,1],[1,1,0,0,0]]);
```

$$AA := \begin{bmatrix} 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 \end{bmatrix} \quad (5.1)$$

```
> with(LinearAlgebra):
  A2:=Transpose(AA);
```

$$A2 := \begin{bmatrix} 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \quad (5.2)$$

```
> A3:=Matrix(5,5):
  for i from 1 to 5 do
  S:=0;
   for j from 1 to 5 do
    S:=S+A2[j,i];
   end do;
   for j from 1 to 5 do
    A3[j,i]:=A2[j,i]/S;
   end do;
  end do:
  A3;
```

$$\begin{bmatrix} 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ \frac{1}{3} & 0 & 0 & 0 & \frac{1}{2} \\ \frac{1}{3} & \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ \frac{1}{3} & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 \end{bmatrix}$$

```
> V1:=evalf(A3.A3.A3.Vector([1/5,1/5,1/5,1/5,1/5]));
```

$$V1 := \begin{bmatrix} 0.1250000000 \\ 0.1111111111 \\ 0.2694444444 \\ 0.3277777778 \\ 0.1666666667 \end{bmatrix}$$

```
> # PageRankは4,3,5,1,2なんですよ．5が上位に来るのが不思議ですが．．．
  正しそう．以下は固有ベクトルを用いた別解．evalf(*,2)で見やすく表示させ
  ています．
> l,v:=evalf(Eigenvectors(A3),2);
```

$$l, v := \begin{bmatrix} 1. \\ 0.09 + 0.51\,I \\ 0.09 - 0.51\,I \\ -0.35 \\ -0.83 \end{bmatrix}, \begin{bmatrix} 0.90 & -0.58 - 0.81\,I & -0.58 + 0.81\,I & -3.2 & -0.34 \\ 0.80 & -0.34 - 0.76\,I & -0.34 + 0.76\,I & 1.2 & -0.60 \\ 1.7 & -0.17 + 0.39\,I & -0.17 - 0.39\,I & 1.9 & -0. \\ 2. & 0.20 + 1.0\,I & 0.20 - 1.0\,I & -0.87 & -1.3 \\ 1. & 1. & 1. & 1. & 1. \end{bmatrix}$$

```
> Column(v,1)*0.16667;
```

$$\begin{bmatrix} 0.150003000000000 \\ 0.133336000000000 \\ 0.283339000000000 \\ 0.333340000000000 \\ 0.166670000000000 \end{bmatrix} \quad (5.3)$$