数式処理実習試験問題，西谷@関西学院大・理工，2015/7/3 実施

1(1)

```
> restart;
  simplify(diff(sqrt(x^2+1)*(x^3+1)^(1/3),x));
```

$$\frac{x\left(2x^3+x+1\right)}{\sqrt{x^2+1}\left(x^3+1\right)^{2/3}} \tag{2.1}$$

1(2)

```
> restart;
  x:=t->a*cos(t)^3;
  y:=t->a*sin(t)^3;
```

$$x := t \to a\cos(t)^3$$
$$y := t \to a\sin(t)^3 \tag{3.1}$$

```
> dxdt:=diff(x(t),t);
  dydt:=diff(y(t),t);
```

$$dxdt := -3a\cos(t)^2\sin(t)$$
$$dydt := 3a\sin(t)^2\cos(t) \tag{3.2}$$

```
> dydt/dxdt;
```

$$-\frac{\sin(t)}{\cos(t)} \tag{3.3}$$

2(1)

```
> restart;
  eq1:=1/(cos(x)^2+4*sin(2)^2);
```

$$eq1 := \frac{1}{\cos(x)^2+4\sin(2)^2} \tag{4.1}$$

```
> diff(eq1,x);
```

$$\frac{2\cos(x)\sin(x)}{\left(\cos(x)^2+4\sin(2)^2\right)^2} \tag{4.2}$$

2(2)

```
> restart;
  eq2:=1/((y-x)^(1/3));
```

$$eq2 := \frac{1}{(y-x)^{1/3}} \tag{5.1}$$

```
> eq3:=int(int(eq2,x=0..y),y=1/n..1);
```

$$eq3 := -\frac{9}{10}\left(\frac{1}{n}\right)^{5/3}+\frac{9}{10} \tag{5.2}$$

```
> limit(eq3,n=infinity);
```

$$\frac{9}{10} \tag{5.3}$$

3(1)

```
> restart;
  with(LinearAlgebra):
> A:=Matrix([[1,-1,1],[-7,2,1],[2,1,2]]);
```

$$A := \begin{bmatrix} 1 & -1 & 1 \\ -7 & 2 & 1 \\ 2 & 1 & 2 \end{bmatrix} \tag{6.1}$$

```
> l,P:=Eigenvectors(A);
```

$$l, P := \begin{bmatrix} 3 \\ -2 \\ 4 \end{bmatrix}, \begin{bmatrix} 0 & -1 & -1 \\ 1 & -2 & 4 \\ 1 & 1 & 1 \end{bmatrix} \tag{6.2}$$

```
> MatrixInverse(P).A.P;
```

$$\begin{bmatrix} 3 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 4 \end{bmatrix} \tag{6.3}$$

3(2)

```
> restart;
  with(LinearAlgebra):
> A:=Matrix([[0,c,b],[c,0,a],[b,a,0]]);
```

$$A := \begin{bmatrix} 0 & c & b \\ c & 0 & a \\ b & a & 0 \end{bmatrix} \tag{7.1}$$

```
> B:=Matrix([[0,1,1],[1,0,1],[1,1,0]]);
```

$$B := \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} \tag{7.2}$$

```
> Determinant(A.B);
```

$$4abc \tag{7.3}$$

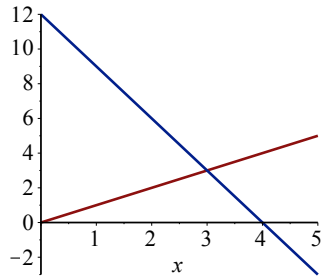
4(1)

```
> restart;
  f1:=x->x;
  f2:=x->-3*x+12;
```

$$f1 := x \to x$$

$$f2 := x \to -3\,x + 12 \qquad (8.1)$$

```
> plot([f1(x),f2(x)],x=0..5);
```




```
> eq1:=int(f1(t),t=0..x);
  g1:=unapply(eq1,x);
```

$$eq1 := \frac{1}{2}\,x^2$$

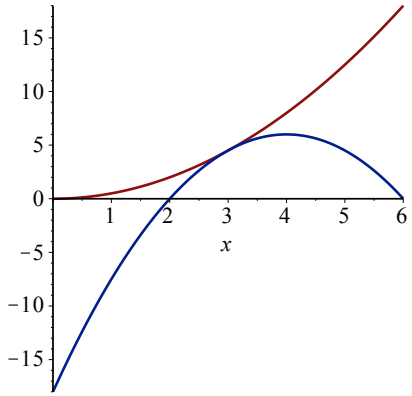
$$g1 := x \to \frac{1}{2}\,x^2 \qquad (8.2)$$

```
> eq2:=int(f2(t),t=3..x)+g1(3);
  g2:=unapply(eq2,x);
```

$$eq2 := -\frac{3}{2}\,x^2 - 18 + 12\,x$$

$$g2 := x \to -\frac{3}{2}\,x^2 - 18 + 12\,x \qquad (8.3)$$

```
> plot([g1(x),g2(x)],x=0..6);
```




```
> a_l:=unapply(diff(g1(x),x),x);
```

$$a\_l := x \to x \qquad (8.4)$$

```
> eq_L:=expand(a_l(a)*(x-a)+g1(a));
```

$$eq\_L := -\frac{1}{2}\,a^2 + a\,x \qquad (8.5)$$

```
> y_L:=unapply(eq_L,x);
```

$$y\_L := x \to -\frac{1}{2}\,a^2 + a\,x \qquad (8.6)$$

```
> Q_x:=solve(y_L(x)=0,x);
```

$$Q\_x := \frac{1}{2}\,a \qquad (8.7)$$

```
> Q:=Vector([Q_x,0]);
```

$$Q := \begin{bmatrix} \frac{1}{2}\,a \\ 0 \end{bmatrix} \qquad (8.8)$$

```
> eq3:=solve(y_L(x)=g2(x),x);
```

$$eq3 := -a + 6,\ \frac{1}{3}\,a + 2 \qquad (8.9)$$

```
> R_x:=eq3[1];
```

$$R\_x := -a + 6 \qquad (8.10)$$

```
> R_y:=expand(g2(R_x));
```

$$R\_y := -\frac{3}{2}\,a^2 + 6\,a \qquad (8.11)$$

```
> R:=Vector([R_x,R_y]);
```

(8.12)

$$R := \begin{bmatrix} -a+6 \\ -\frac{3}{2}a^2+6a \end{bmatrix} \quad (8.12)$$

```
> H:=Vector([R_x,0]);
```

$$H := \begin{bmatrix} -a+6 \\ 0 \end{bmatrix} \quad (8.13)$$

```
> eq4:=expand((Q_x-R_x)*R_y/2);
```

$$eq4 := -\frac{9}{8}a^3+9a^2-18a \quad (8.14)$$

```
> S:=unapply(eq4,a);
```

$$S := a \to -\frac{9}{8}a^3+9a^2-18a \quad (8.15)$$

```
> s2:=solve(diff(S(a),a),a);
```

$$s2 := \frac{4}{3}, 4 \quad (8.16)$$

```
> a_max:=s2[1];
```

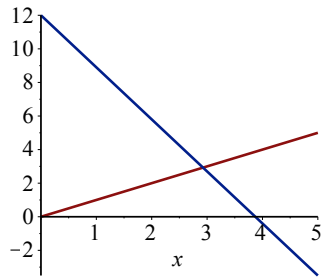
$$a\_max := \frac{4}{3} \quad (8.17)$$

4(2)

```
> restart;
  f1:=x->x;
  f2:=x->-3.1*x+12;
```

$$f1 := x \to x$$
$$f2 := x \to -3.1x+12 \quad (9.1)$$

```
> plot([f1(x),f2(x)],x=0..5);
```




```
> x0:=solve(f1(x)=f2(x),x);
```

$$x0 := 2.926829268 \quad (9.2)$$

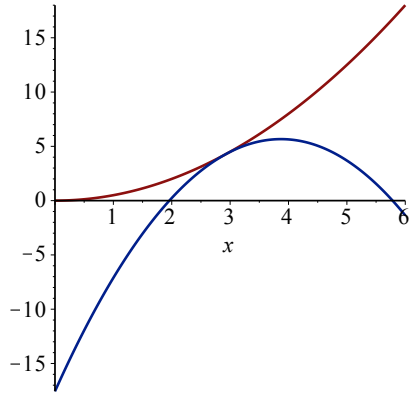
```
> eq1:=int(f1(t),t=0..x);
  g1:=unapply(eq1,x);
```

$$eq1 := \frac{1}{2}x^2$$
$$g1 := x \to \frac{1}{2}x^2 \quad (9.3)$$

```
> eq2:=int(f2(t),t=x0..x)+g1(x0);
  g2:=unapply(eq2,x);
```

$$eq2 := -1.550000000x^2-17.56097561+12.x$$
$$g2 := x \to -1.550000000x^2-17.56097561+12.x \quad (9.4)$$

```
> plot([g1(x),g2(x)],x=0..6);
```




```
> a_l:=unapply(diff(g1(x),x),x);
```

$$a\_l := x \to x \quad (9.5)$$

```
> eq_L:=expand(a_l(a)*(x-a)+g1(a));
```

$$eq\_L := -\frac{1}{2}a^2+ax \quad (9.6)$$

```
> y_L:=unapply(eq_L,x);
```

$$y\_L := x \to -\frac{1}{2}a^2+ax \quad (9.7)$$

```
> Q_x:=solve(y_L(x)=0,x);
```

$$Q\_x := \frac{1}{2}a \quad (9.8)$$

```
> eq3:=solve(y_L(x)=g2(x),x);
```

$$eq3 := -0.3225806452a+3.870967742+0.000006451612903\sqrt{1.025000000\,10^{10}a^2-6.000000000\,10^{10}a+8.780487804\,10^{10}}, -0.3225806452a+3.870967742-0.000006451612903 \quad (9.9)$$

$$\sqrt{1.025000000\,10^{10}\,a^2 - 6.000000000\,10^{10}\,a + 8.780487804\,10^{10}}$$

```
> R_x:=eq3[1];
```

$$R\_x := -0.3225806452\,a + 3.870967742 + 0.000006451612903\,\sqrt{1.025000000\,10^{10}\,a^2 - 6.000000000\,10^{10}\,a + 8.780487804\,10^{10}} \quad (9.10)$$

```
> R_y:=expand(g2(R_x));
```

$$R\_y := -0.8225806453\,a^2 + 3.870967743\,a + 0.000006451612904\,a\,\sqrt{1.025000000\,10^{10}\,a^2 - 6.000000000\,10^{10}\,a + 8.780487804\,10^{10}} - 1.\,10^{-8} + 1.\,10^{-14}\,\sqrt{1.025000000\,10^{10}\,a^2 - 6.000000000\,10^{10}\,a + 8.780487804\,10^{10}} \quad (9.11)$$

```
> R:=Vector([R_x,R_y]);
```

$$R := \left[\left[-0.3225806452\,a + 3.870967742 + 0.000006451612903\,\sqrt{1.025000000\,10^{10}\,a^2 - 6.000000000\,10^{10}\,a + 8.780487804\,10^{10}}\right], \left[-0.8225806453\,a^2 + 3.870967743\,a + 0.000006451612904\,a\,\sqrt{1.025000000\,10^{10}\,a^2 - 6.000000000\,10^{10}\,a + 8.780487804\,10^{10}} - 1.\,10^{-8} + 1.\,10^{-14}\,\sqrt{1.025000000\,10^{10}\,a^2 - 6.000000000\,10^{10}\,a + 8.780487804\,10^{10}}\right]\right] \quad (9.12)$$

```
> H:=Vector([R_x,0]);
```

$$H := \left[\left[-0.3225806452\,a + 3.870967742 + 0.000006451612903\,\sqrt{1.025000000\,10^{10}\,a^2 - 6.000000000\,10^{10}\,a + 8.780487804\,10^{10}}\right], [0]\right] \quad (9.13)$$

```
> eq4:=expand((Q_x-R_x)*R_y/2);
```

$$eq4 := -0.5516389180\,a^3 + 4.432882415\,a^2 + 0.000005306971905\,a^2\,\sqrt{1.025000000\,10^{10}\,a^2 - 6.000000000\,10^{10}\,a + 8.780487804\,10^{10}} - 9.319560420\,a - 0.00002497398544\,a\,\sqrt{1.025000000\,10^{10}\,a^2 - 6.000000000\,10^{10}\,a + 8.780487804\,10^{10}} + 1.652242329\,10^{-8} + 1.290322580\,10^{-14}\,\sqrt{1.025000000\,10^{10}\,a^2 - 6.000000000\,10^{10}\,a + 8.780487804\,10^{10}} \quad (9.14)$$

$$\sqrt{1.025000000\,10^{10}\,a^2 - 6.000000000\,10^{10}\,a + 8.780487804\,10^{10}}$$

```
> S:=unapply(eq4,a);
```

$$S := a \rightarrow -0.5516389180\,a^3 + 4.432882415\,a^2 + 0.000005306971905\,a^2\,\sqrt{1.025000000\,10^{10}\,a^2 - 6.000000000\,10^{10}\,a + 8.780487804\,10^{10}} - 9.319560420\,a - 0.00002497398544\,a\,\sqrt{1.025000000\,10^{10}\,a^2 - 6.000000000\,10^{10}\,a + 8.780487804\,10^{10}} + 1.652242329\,10^{-8} + 1.290322580\,10^{-14}\,\sqrt{1.025000000\,10^{10}\,a^2 - 6.000000000\,10^{10}\,a + 8.780487804\,10^{10}} \quad (9.15)$$

```
> s2:=solve(diff(S(a),a),a);
```

$$s2 := 1.306155359, 3.855134992, 11.56540468 \quad (9.16)$$

```
> a_max:=evalf(s2[1]);
```

$$a\_max := 1.306155359 \quad (9.17)$$

```
> evalf(x0);
```

$$2.926829268 \quad (9.18)$$