▼ 1(a)

```
> restart;
  diff(sqrt((x^2-1)/(x^2+1)),x);
```

$$\frac{1}{2}\frac{\dfrac{2x}{x^2+1}-\dfrac{2\left(x^2-1\right)x}{\left(x^2+1\right)^2}}{\sqrt{\dfrac{x^2-1}{x^2+1}}} \qquad (1.1)$$

▼ 1(b)

```
> assume(x>0);
  I1:=int(log(x/y^2),y=1..x);
```

$$I1 := -\ln(x\sim) - 2 - \ln(x\sim)\,x\sim + 2\,x\sim \qquad (2.1)$$

```
> int(I1,x=1..2);
```

$$\frac{11}{4} - 4\ln(2) \qquad (2.2)$$

▼ 2(a)

```
> restart;
  with(LinearAlgebra):
> A:=Matrix([[1,2,3],[2,1,3],[3,3,2]]);
```

$$A := \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 3 & 3 & 2 \end{bmatrix} \qquad (3.1)$$

```
> E:=IdentityMatrix(3);
```

$$E := \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad (3.2)$$

```
> Determinant(A-t*E);
```

$$12 + 17\,t + 4\,t^2 - t^3 \qquad (3.3)$$

```
> solve(Determinant(A-t*E)=0,t);
```

$$-1,\ \frac{5}{2} - \frac{1}{2}\sqrt{73},\ \frac{5}{2} + \frac{1}{2}\sqrt{73} \qquad (3.4)$$

▼ 2(b)

```
> restart;
  with(LinearAlgebra):
> Q:=Matrix([[3,2],[2,6]]);
  xx:=Vector([x,y]);
  bb:=Vector([-6,-2]);
```

$$Q := \begin{bmatrix} 3 & 2 \\ 2 & 6 \end{bmatrix}$$

$$xx := \begin{bmatrix} x \\ y \end{bmatrix}$$

$$bb := \begin{bmatrix} -6 \\ -2 \end{bmatrix} \qquad (4.1)$$

```
> expand(Transpose(xx).Q.xx+Transpose(bb).xx+2);
```

$$3\,x^2 + 4\,xy + 6\,y^2 - 6\,x - 2\,y + 2 \qquad (4.2)$$

```
> l,V:=Eigenvectors(Q);
```

$$l, V := \begin{bmatrix} 7 \\ 2 \end{bmatrix}, \begin{bmatrix} \dfrac{1}{2} & -2 \\ 1 & 1 \end{bmatrix} \qquad (4.3)$$

```
> v1:=Normalize(Column(V,1),Euclidean);
  v2:=Normalize(Column(V,2),Euclidean);
```

$$v1 := \begin{bmatrix} \dfrac{1}{5}\sqrt{5} \\ \dfrac{2}{5}\sqrt{5} \end{bmatrix}$$

$$v2 := \begin{bmatrix} -\dfrac{2}{5}\sqrt{5} \\ \dfrac{1}{5}\sqrt{5} \end{bmatrix} \qquad (4.4)$$

```
> P:=<v1|v2>;
```

$$P := \begin{bmatrix} \dfrac{1}{5}\sqrt{5} & -\dfrac{2}{5}\sqrt{5} \\ \dfrac{2}{5}\sqrt{5} & \dfrac{1}{5}\sqrt{5} \end{bmatrix} \qquad (4.5)$$

```
> Transpose(P).Q.P;
```

$$\begin{bmatrix} 7 & 0 \\ 0 & 2 \end{bmatrix} \qquad (4.6)$$

```
> yy:=Vector([xp,yp]);
```

$$yy := \begin{bmatrix} xp \\ yp \end{bmatrix} \qquad (4.7)$$

```
> e1:=expand(Transpose(yy).Transpose(P).Q.P.yy)+Transpose(bb).P.
  yy+2;
```

$$e1 := 7\,xp^2 + 2\,yp^2 - 2\,xp\sqrt{5} + 2\,yp\sqrt{5} + 2 \qquad (4.8)$$

```
> expand(7*(xp-sqrt(5)/7)^2+2*(yp+sqrt(5)/2)^2+2-45/14);
```

$$7\,xp^2 + 2\,yp^2 - 2\,xp\sqrt{5} + 2\,yp\sqrt{5} + 2 \qquad (4.9)$$

3(a)

```
> restart;
  eq1:=(2^x+3^x)*(9/2^x+4/3^x);
```

$$eq1 := \left(2^x + 3^x\right)\left(\frac{9}{2^x} + \frac{4}{3^x}\right) \tag{5.1}$$

```
> expand(eq1);
```

$$13 + \frac{4 \, 2^x}{3^x} + \frac{9 \, 3^x}{2^x} \tag{5.2}$$

```
> expand(13+4/X+9*X);
```

$$13 + \frac{4}{X} + 9\,X \tag{5.3}$$

```
> eq2:=13+4/X+9*X-a;
```
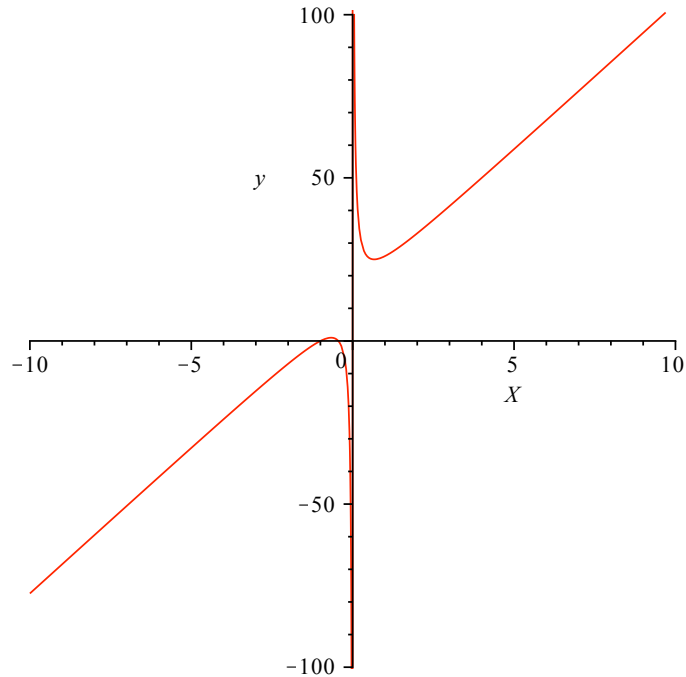
$$eq2 := 13 + \frac{4}{X} + 9\,X - a \tag{5.4}$$

```
> sort(expand(eq2*X),X);
```

$$9\,X^2 + 13\,X - a\,X + 4 \tag{5.5}$$

```
> solve(expand((13-a)^2-4*9*4),a);
```

$$25, 1 \tag{5.6}$$

```
> plot(eq2+a,X,y=-100..100);
```



```
> s1:=solve(expand(eq2*X),X);
```

$$s1 := -\frac{13}{18} + \frac{1}{18}\,a + \frac{1}{18}\sqrt{25 - 26\,a + a^2}\,, \; -\frac{13}{18} + \frac{1}{18}\,a$$
$$- \frac{1}{18}\sqrt{25 - 26\,a + a^2} \tag{5.7}$$

```
> expand(s1[1]*s1[2]);
```

$$\frac{4}{9} \tag{5.8}$$

```
> simplify(solve(4/9=(3/2)^x,x));
```

$$-2 \tag{5.9}$$

3(b)

```
> s2:=solve(subs(a=50,eq2),X);
```

$$s2 := 4, \; \frac{1}{9} \tag{6.1}$$

```
> expand(solve(s2[1]=(3/2)^x,x));
```

$$\frac{2\ln(2)}{\ln(3) - \ln(2)} \tag{6.2}$$

```
> expand(solve(s2[2]=(3/2)^x,x));
```

$$-\frac{2\ln(3)}{\ln(3) - \ln(2)} \tag{6.3}$$